Outline

- Example
- Comparison of Means
- Definition of Power and Power Properties
- Power calculations using web-based tool
- Comparison of Proportions
- Other considerations for planning an experiment and analyzing data
- Confounding
- Extras

Online statistics resources:

CTSpedia:

http://www.ctspedia.org/do/view/CTSpedia

This has links to educational materials and other items (statistical tools, articles…)

Research Aim

- Compare two weight reduction diets (A, B)
  - Which is better?

Better?

- Compare weight reductions on both diets
- Compare sustained weight loss for both diets
- Compare times to achieve certain weight loss
- Compare compliance
- Compare reduction in LDL
- Compare success proportions

Which one results in the greatest weight reduction?

Hypothesize:

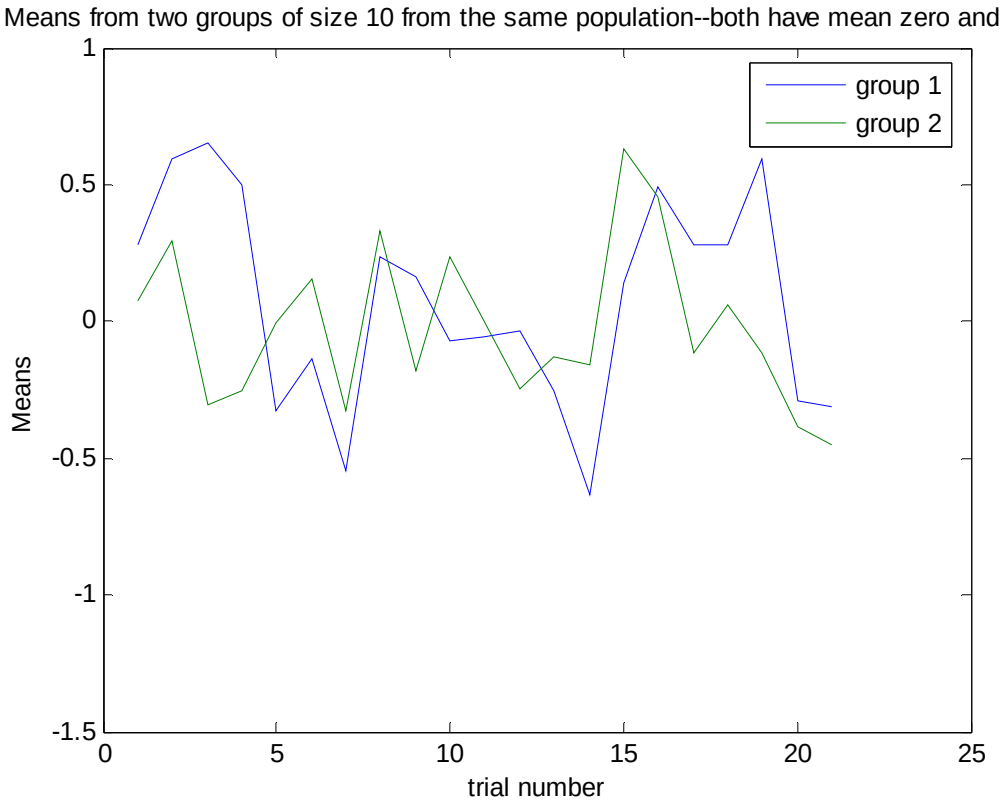
New weight reduction diet results in greater weight reduction after 1 year on the average than standard weight reduction diet

Experiment

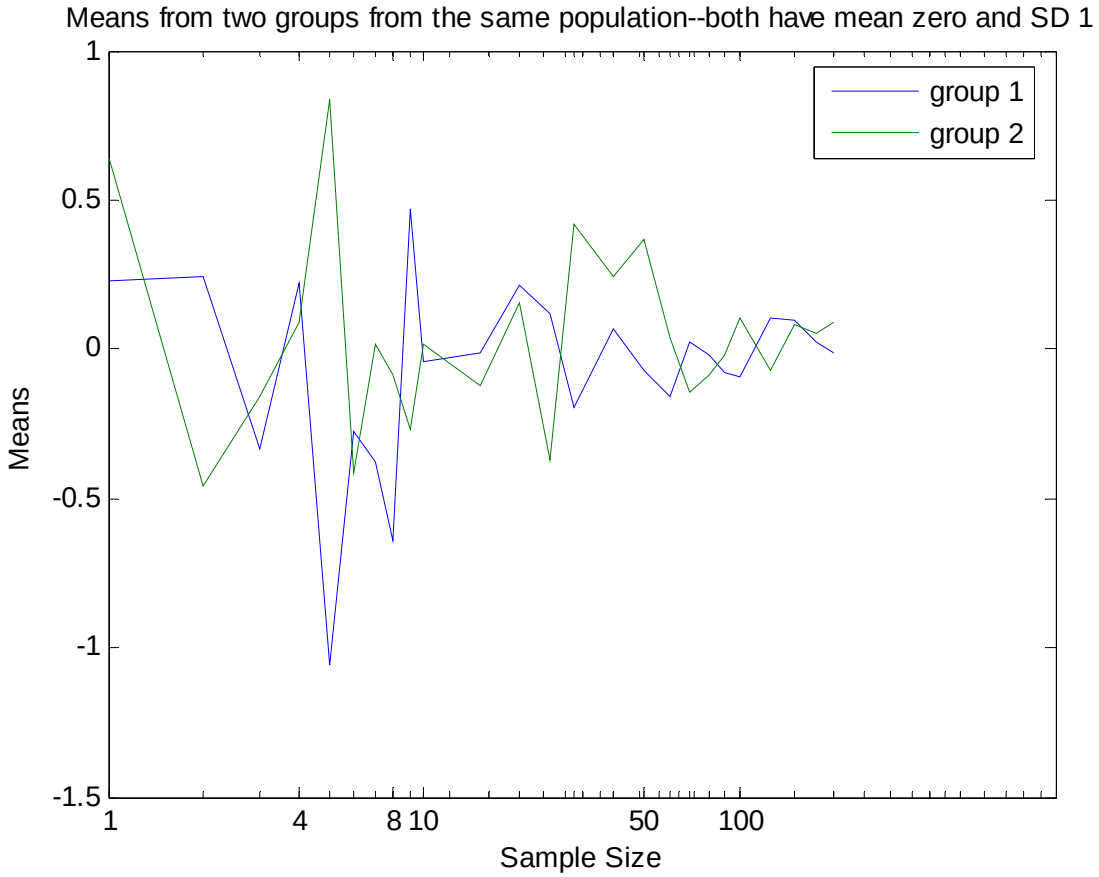
Take 20 people—randomly assign 10 to standard program, 10 to new program

Compare mean weights after 1 year for each program?

Compare mean reductions after 1 year for each program? (Other options as well.)


Means from two groups of size 10 from the same population--both have mean zero and SD 1
group 1
group 2
Means
trial number
1
0.5
0
-0.5
-1
-1.5
0
5
10
15
20
25

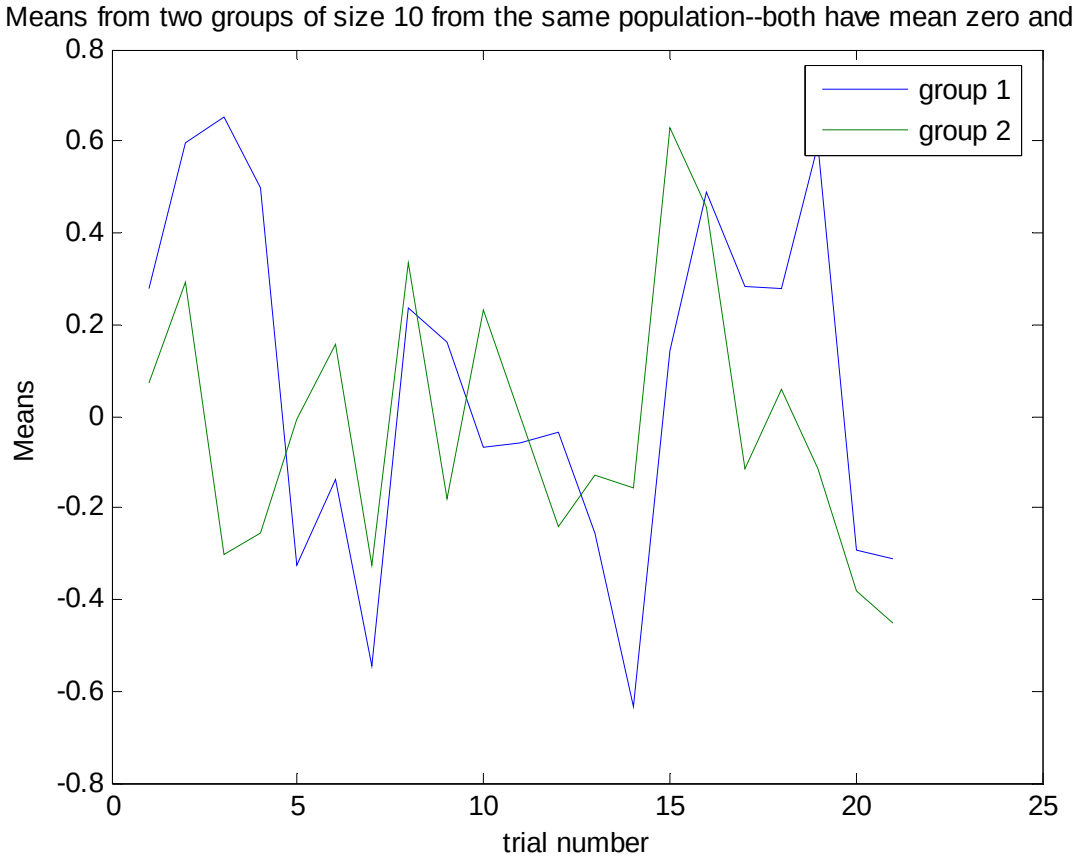
The variability in the sample mean will be less when you have larger sample sizes.



T-statistic
(assumptions required)

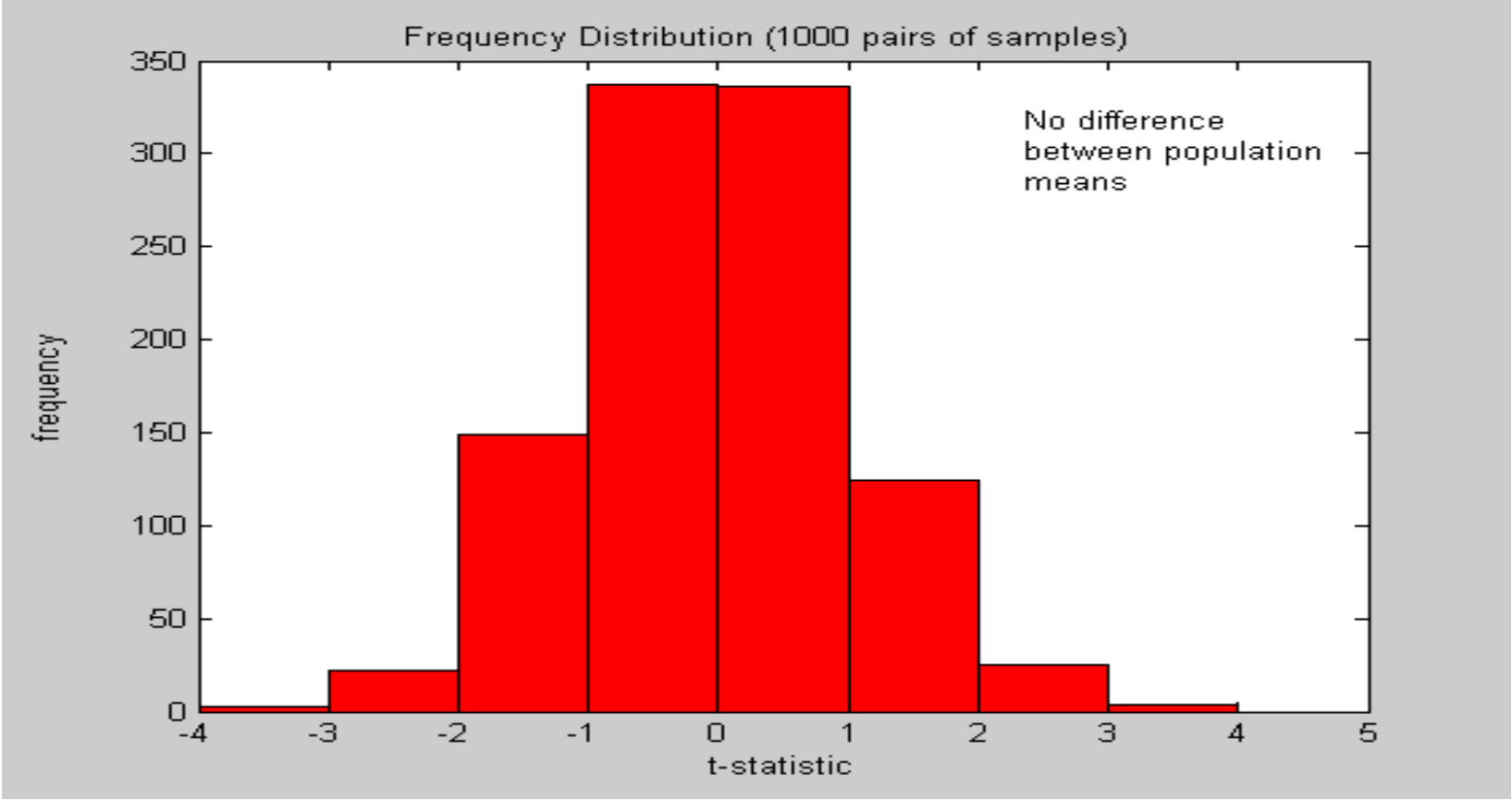
$$t = \frac{\bar{Y}_1 - \bar{Y}_2}{SE}$$

Again, the sample means differ from trial to trial:



The t-tests will also differ from trial to trial.

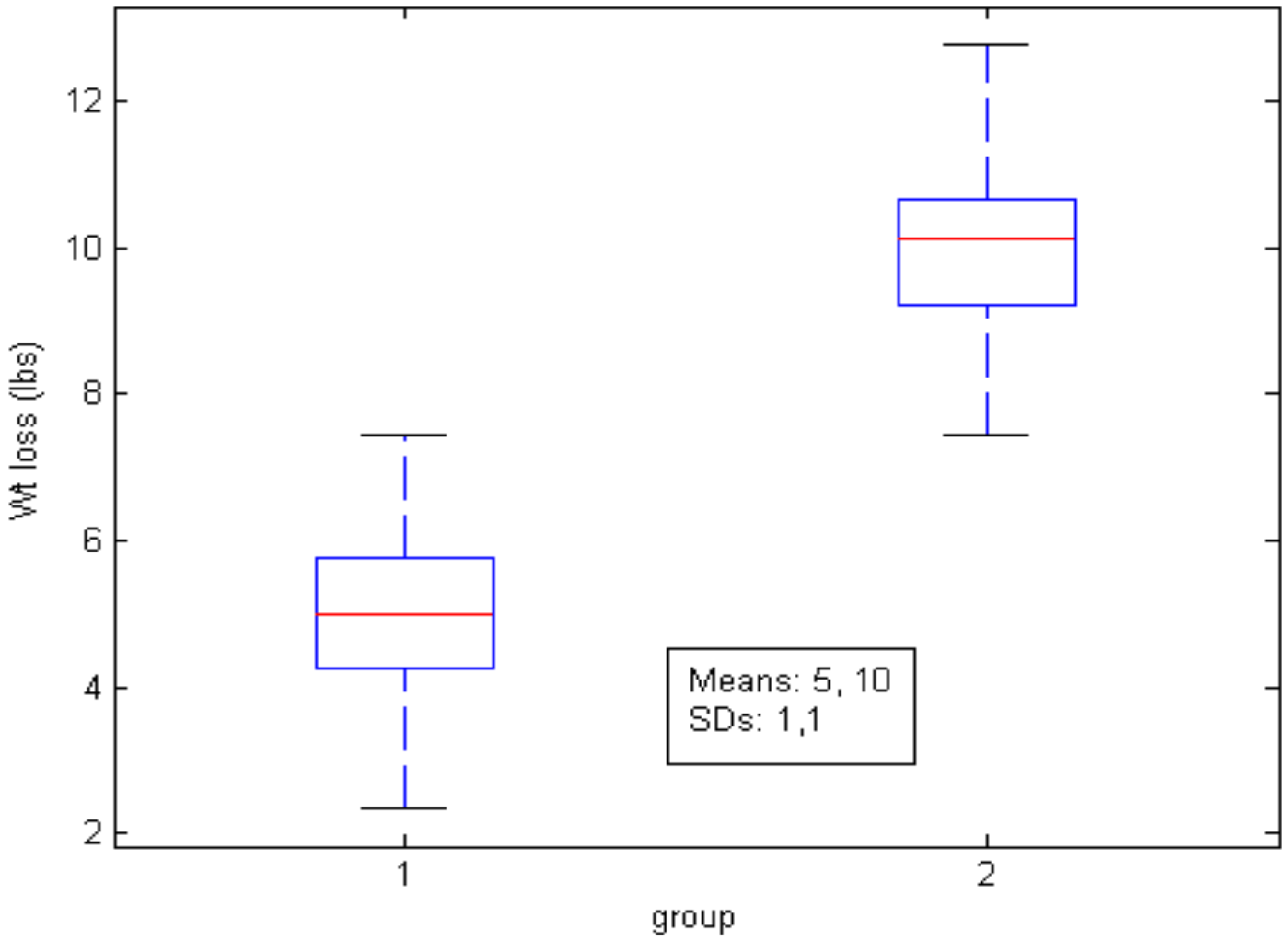
Sample sizes of 10 for each group
No difference between the population means

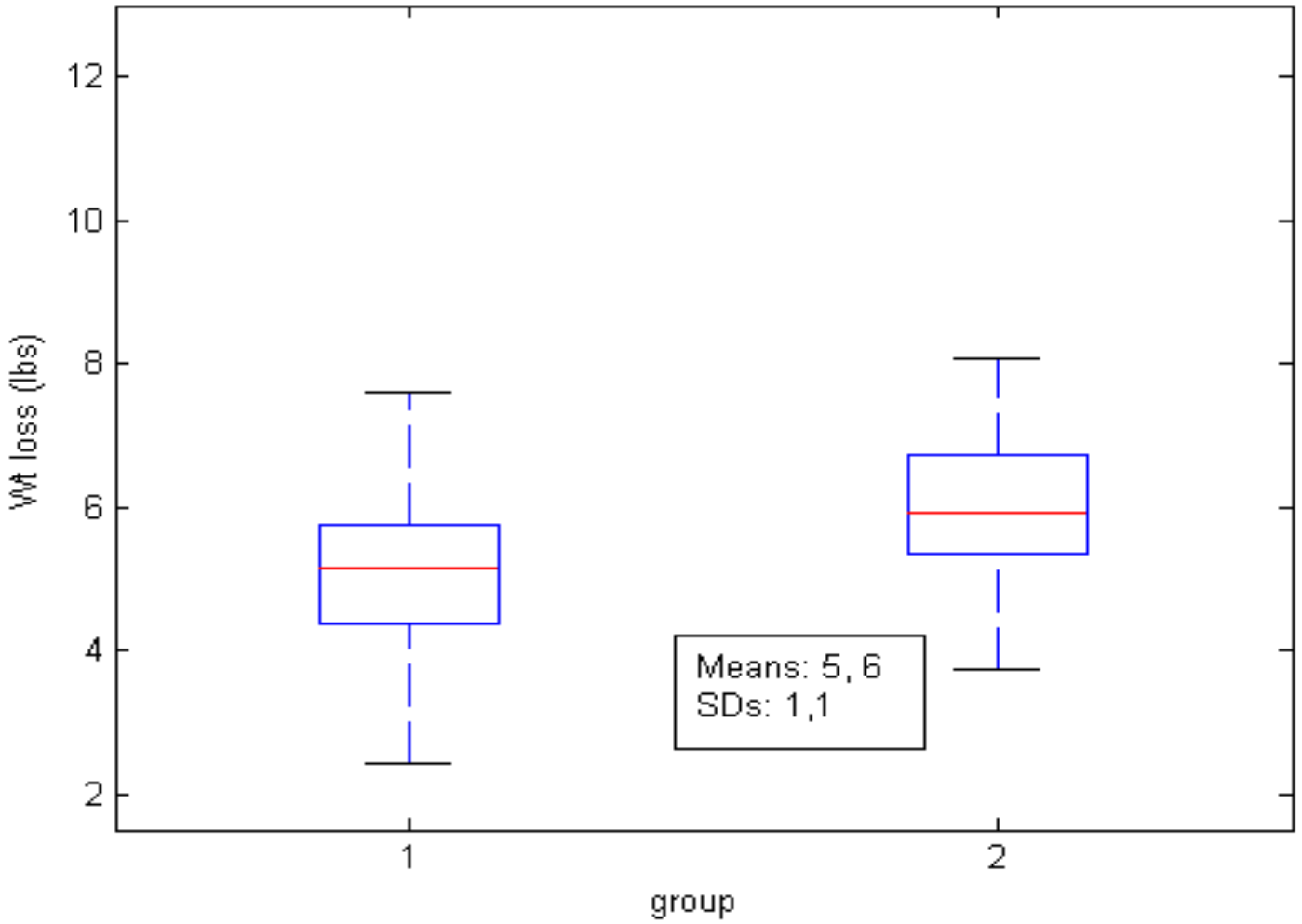


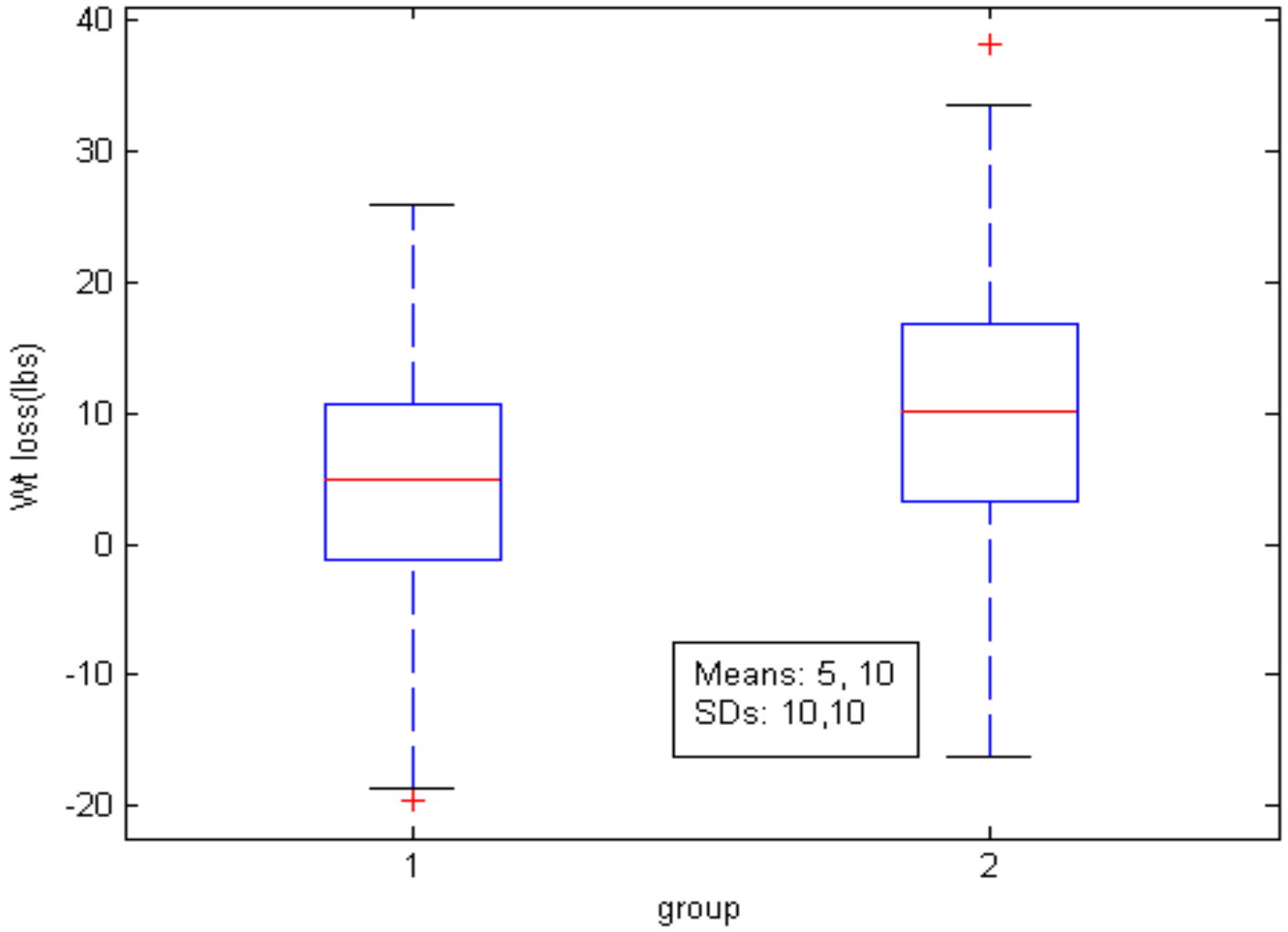
- Conclude there is a difference in mean weight reductions if the t-statistic is large in magnitude which means the p-value is small.
- For power considerations, pick a significance level, e.g .05, then p-value < .05 if

$$t > 2.101 \text{ or } t < -2.101$$

- We set up the test so that the proportion of times we incorrectly conclude there is a difference between the population means is small (Type I error rate: $\alpha = .05$ here)


Wt loss (lbs)
12
10
8
6
4
2
1
2
group
Means: 5, 10
SDs: 1,1


12
10
8
6
4
2
Wt loss (lbs)
1
2
group
Means: 5, 6
SDs: 1,1

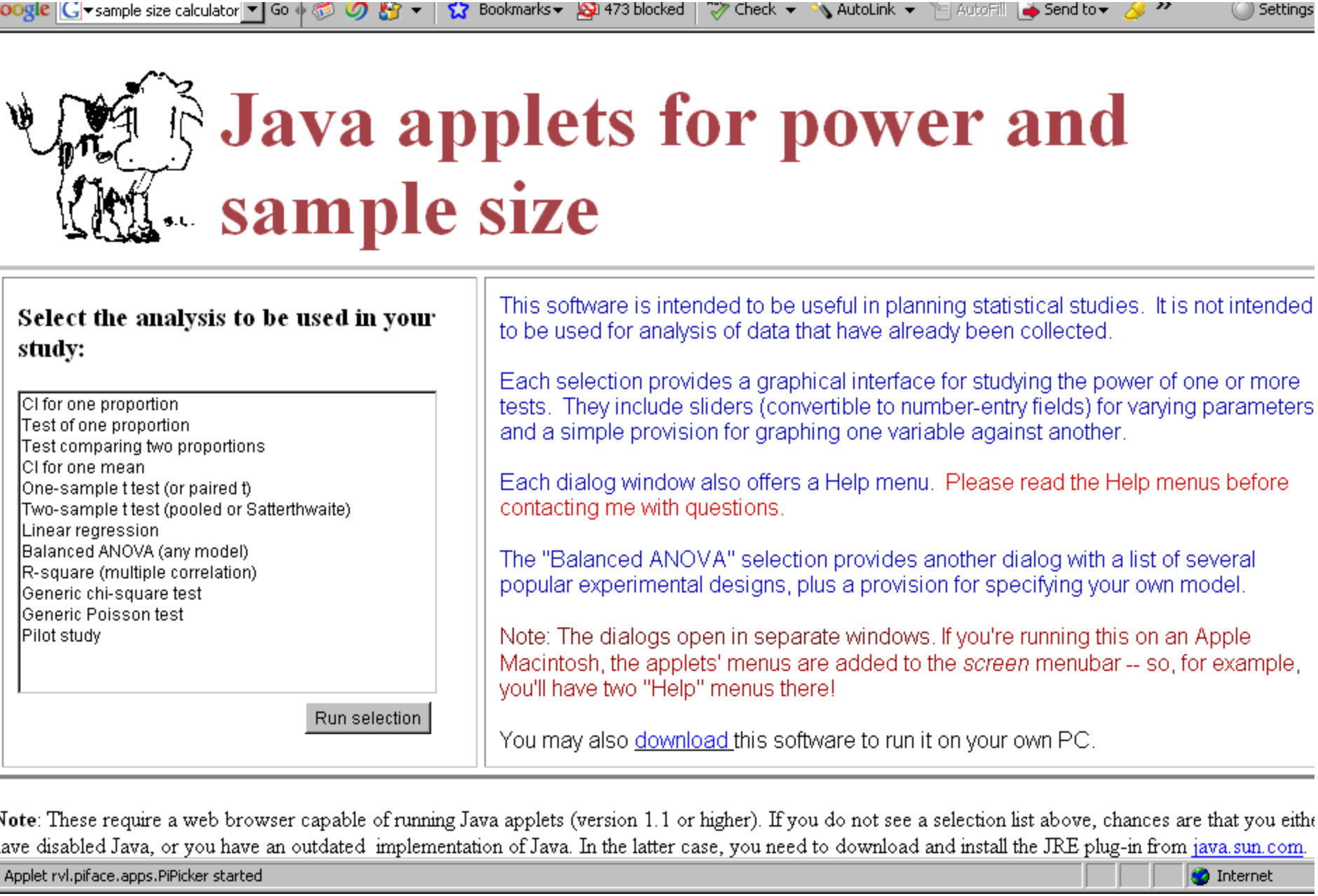

40
30
20
10
0
-10
-20
Wt loss(lbs)
1
2
group
Means: 5, 10
SDs: 10,10

- Power for testing a hypothesis about the difference in means: chance of **concluding** there is a difference between the means when there really is a difference.
- Want power to be high when there is a meaningful difference between the means

- Power for comparing means depends on
  - True difference in the means
  - SD's within each group
  - Sample size
  - Significance level
  - One vs Two-sided Test

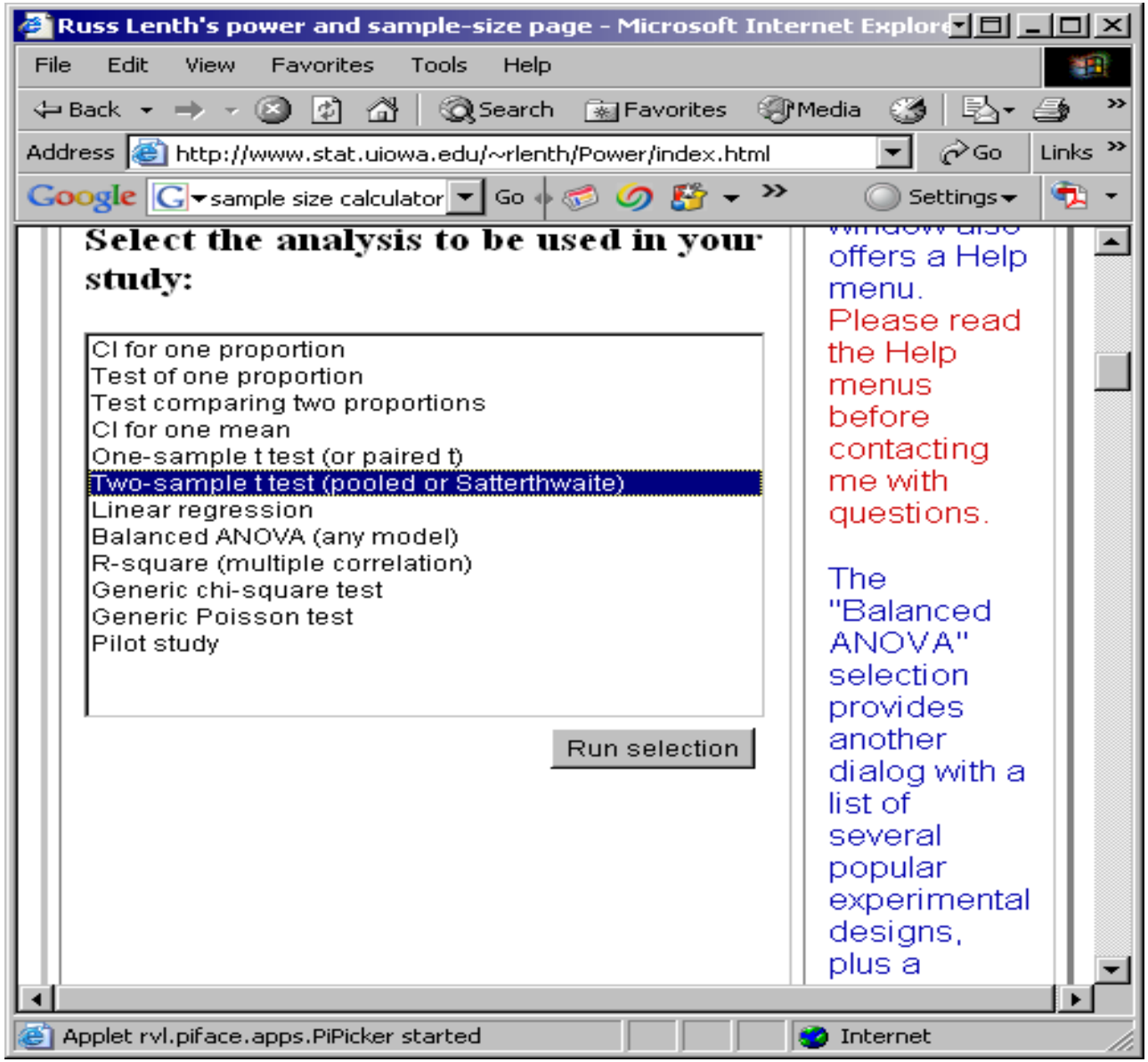
- Examples:

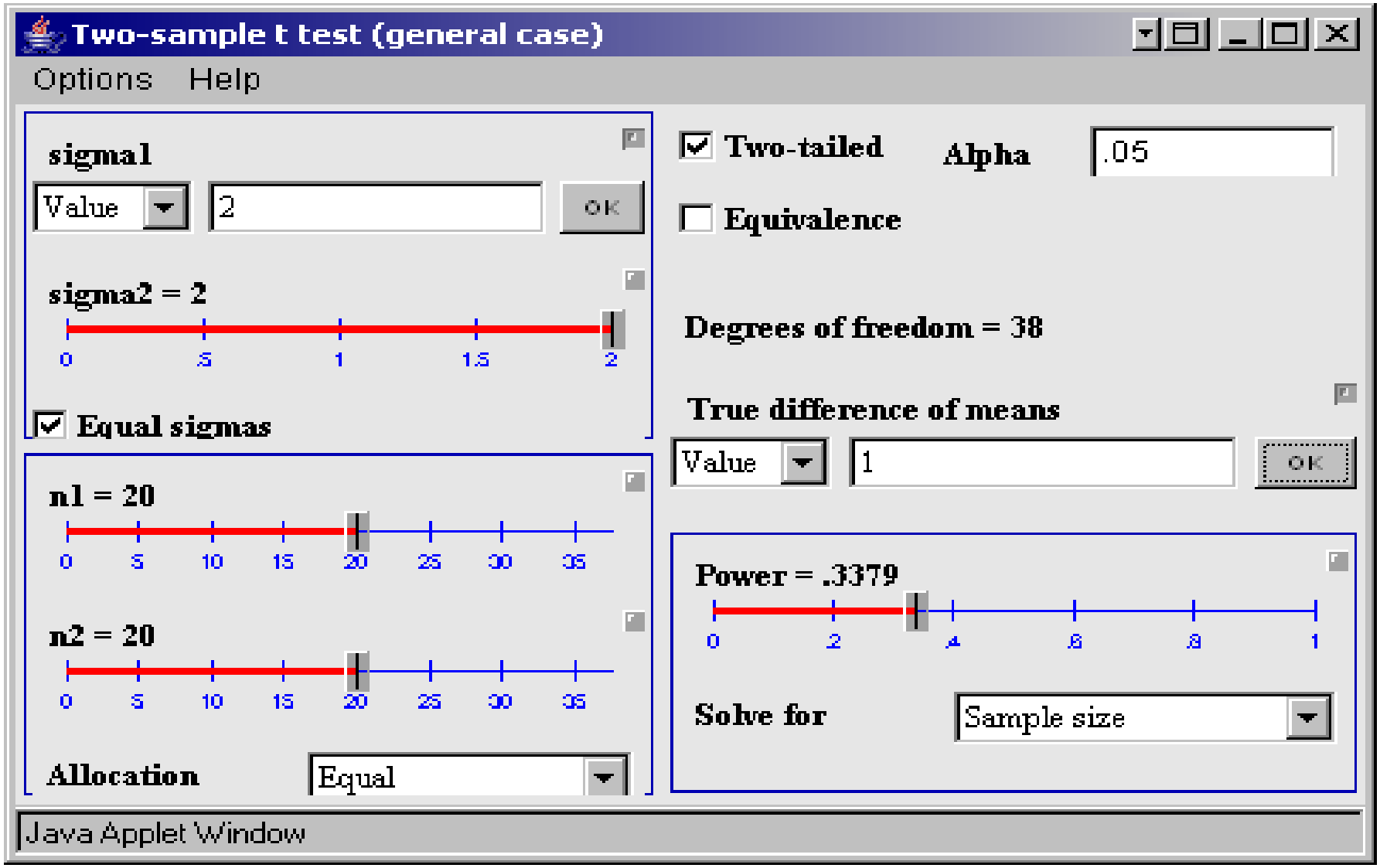
• True means: 5, 7	True means: 5, 6
• SD's: 2,2	SD's: 2, 2
• Sample sizes: 20,20	Sample sizes: 20, 20
• Two-sided test	Two-sided test
• $\alpha = .05$	$\alpha = .05$
Power = 87%	Power=34%



http://homepage.stat.uiowa.edu/~rlenth/Power/index.html

Two-Sample t-test

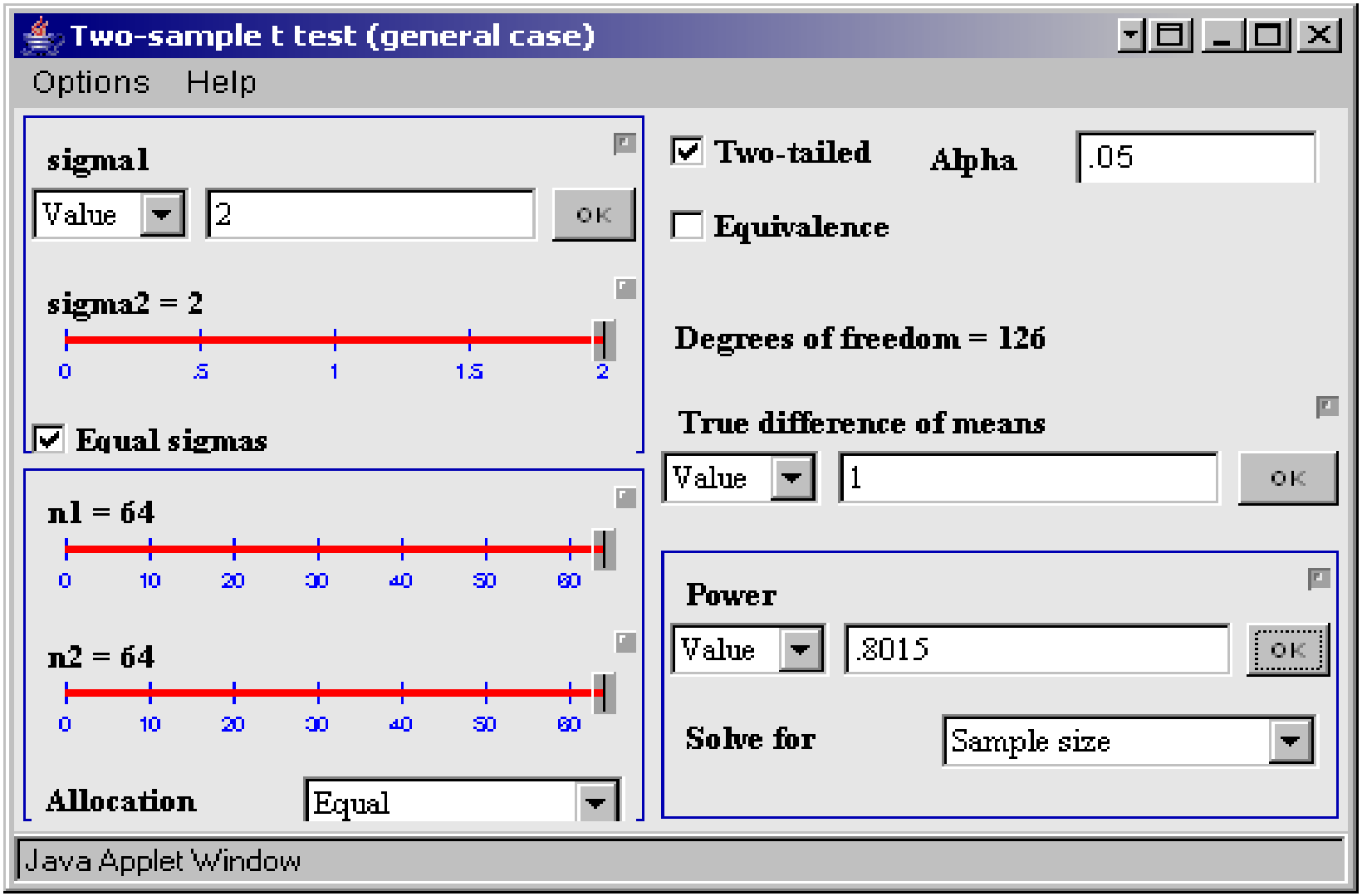



Two-sample t test (general case)
Options
Help
sigma1
Value
2
OK
sigma2 = 2
0
.5
1
1.5
2
Equal sigmas
n1 = 20
0
5
10
15
20
25
30
35
n2 = 20
0
5
10
15
20
25
30
35
Allocation
Equal
Two-tailed
Alpha
.05
Equivalence
Degrees of freedom = 38
True difference of means
Value
1
OK
Power = .3379
0
.2
.4
.6
.8
1
Solve for
Sample size
Java Applet Window

- When planning a study to compare means via a two-sample t-test, typically specify
  - Power
  - Significance level
  - Mean difference want to detect
  - SD's
  - One or two-sided test

  Then

  Find Needed Sample Sizes


Two-sample t test (general case)
Options
Help
sigma1
Value
2
OK
sigma2 = 2
0
.5
1
1.5
2
Equal sigmas
n1 = 64
0
10
20
30
40
50
60
n2 = 64
0
10
20
30
40
50
60
Allocation
Equal
Two-tailed
Alpha
.05
Equivalence
Degrees of freedom = 126
True difference of means
Value
1
OK
Power
Value
.8015
OK
Solve for
Sample size
Java Applet Window

For comparing means:

- Power increases as difference between means increases
- Power increases as SD(s) decreases
- Power increases as sample size increases
- Power increases as significance level α increases

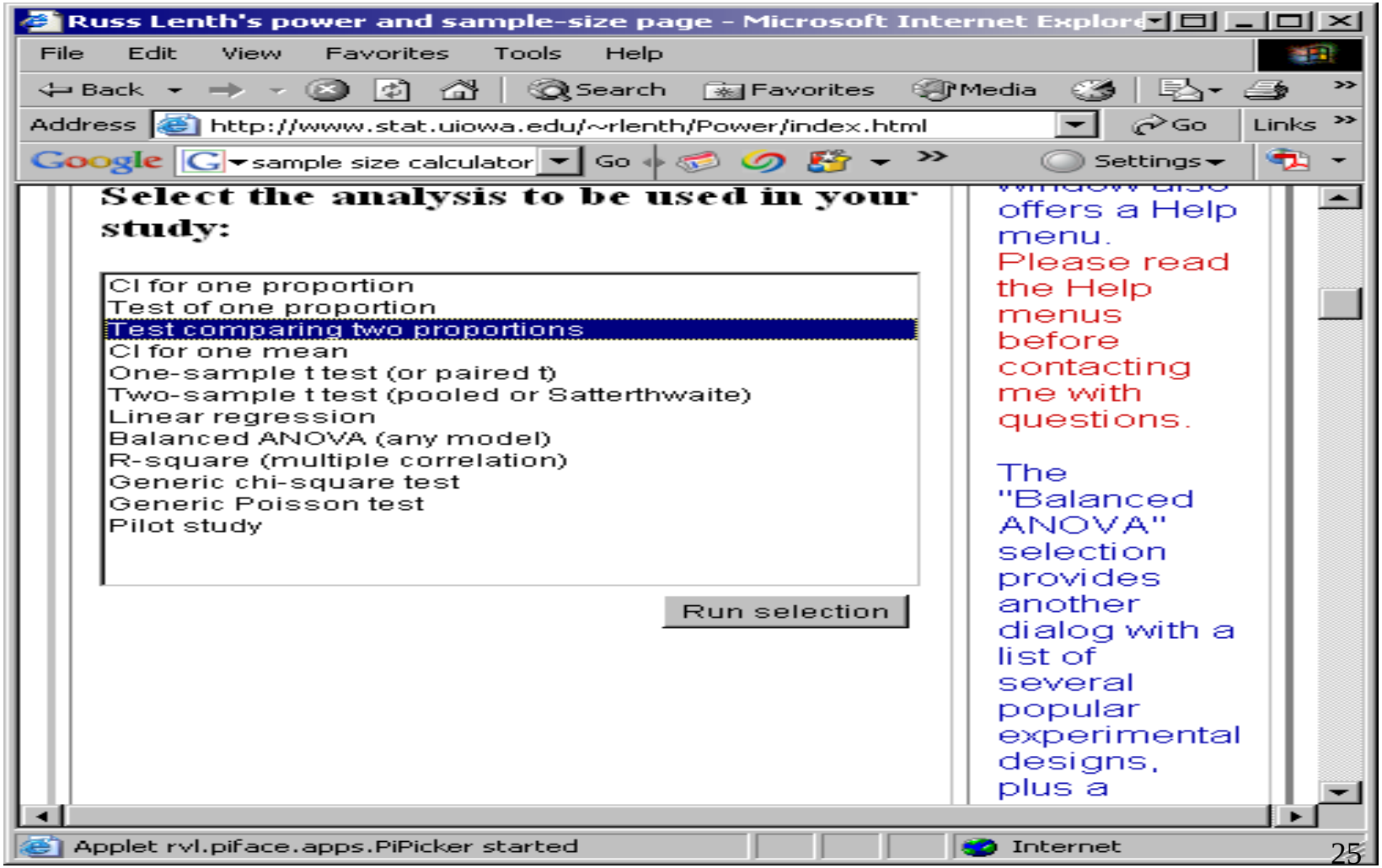
How do you pick some of these quantities?

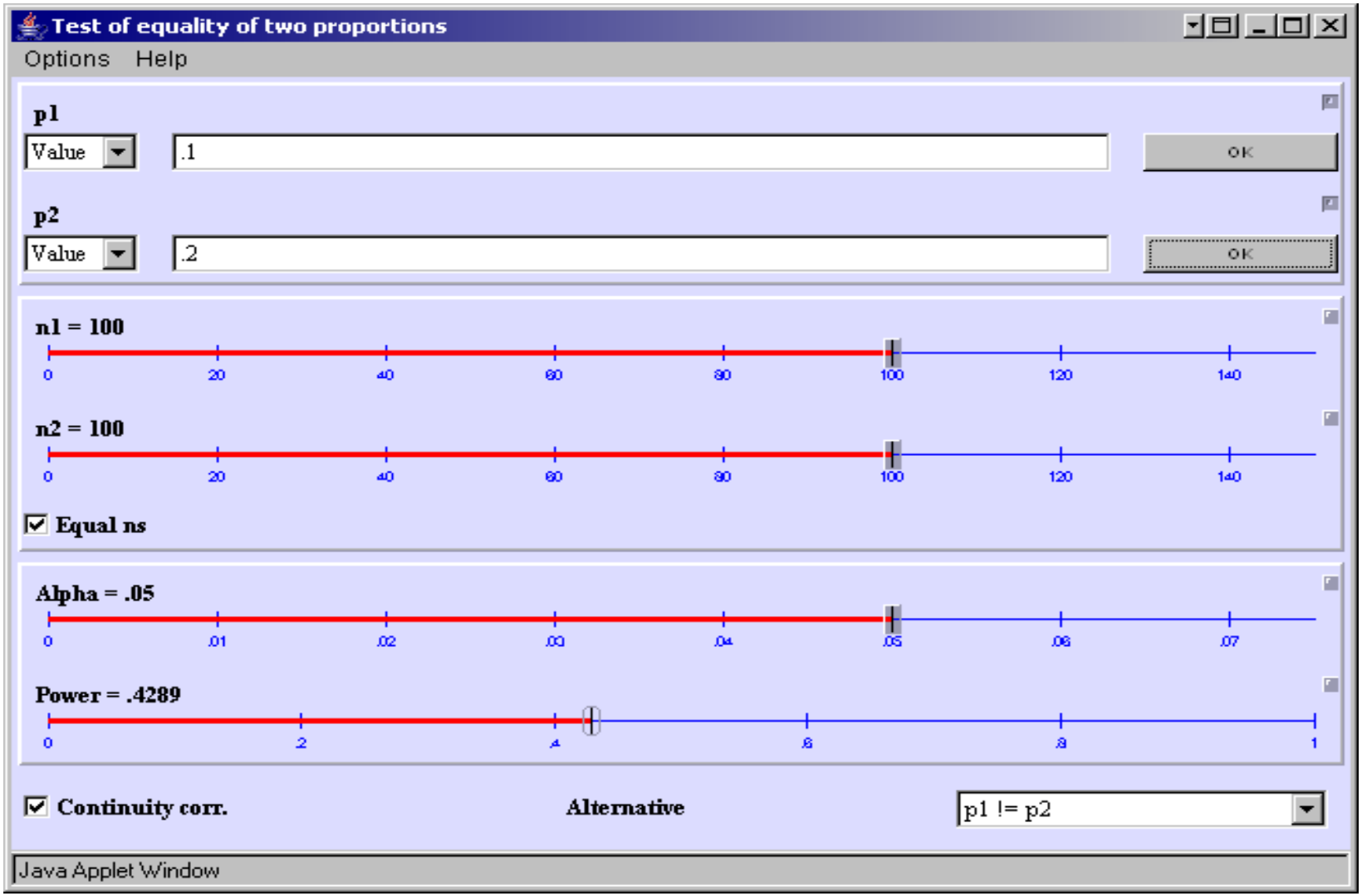
- Difference you want to be able to detect
  - Want to detect difference in mean weight loss of 100 lbs between Atkins and Low-Fat diet after 1 year?
  - Difference of .25 lbs?

  What is the smallest difference that is meaningful?

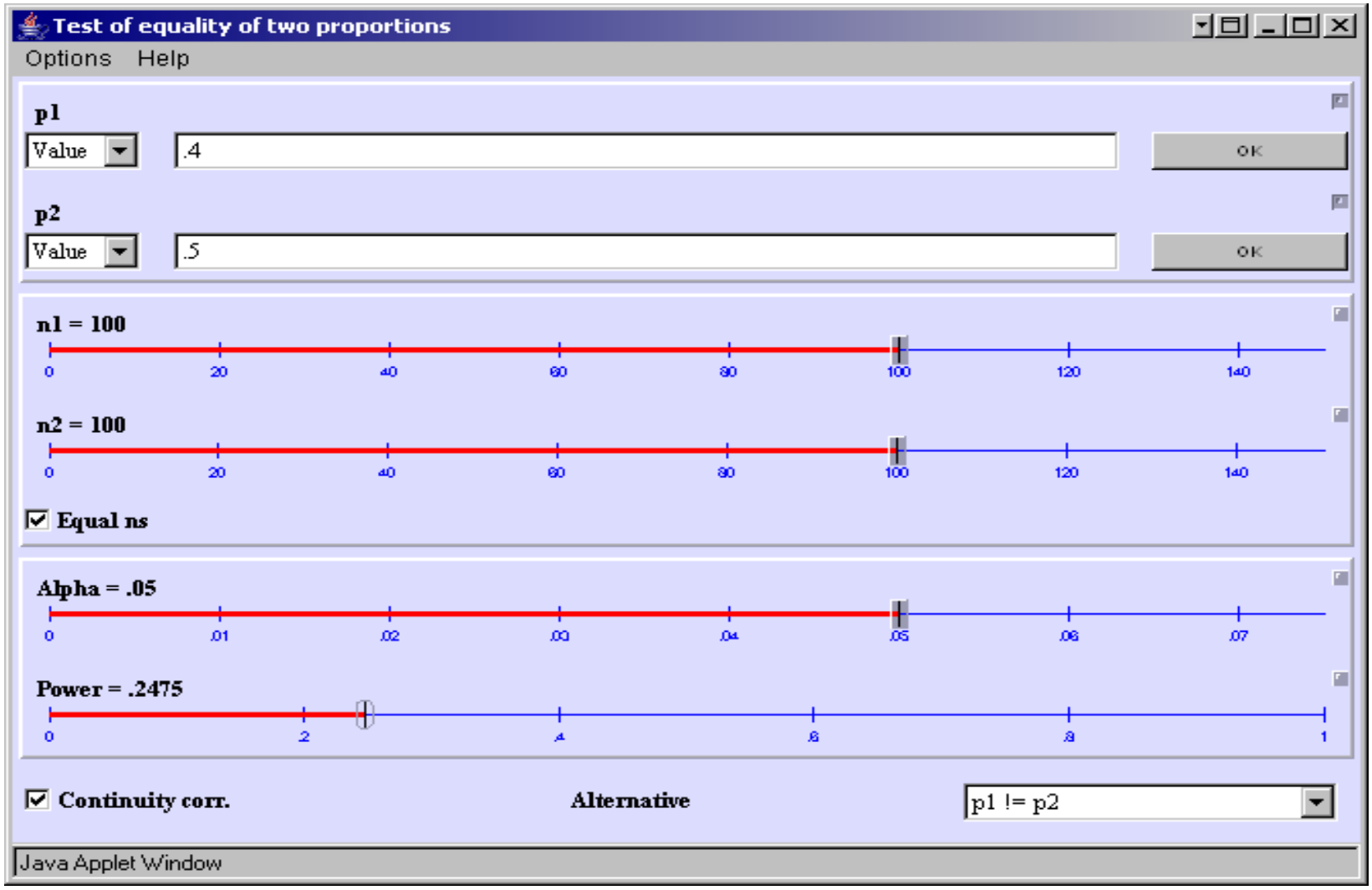
- SD's
  - Pilot data, other studies

Compare proportions




Test of equality of two proportions
Options Help
p1
Value
.1
OK
p2
Value
.2
OK
n1 = 100
0 20 40 60 80 100 120 140
n2 = 100
0 20 40 60 80 100 120 140
Equal ns
Alpha = .05
0 .01 .02 .03 .04 .05 .06 .07
Power = .4289
0 .2 .4 .6 .8 1
Continuity corr.
Alternative
p1 != p2
Java Applet Window

Power =.4289



Power=.2475

Note some differences in different programs an based on options—such as continuity corr, here

- Note that power depends on actual proportions not just the difference—in the example, power greater for comparing .1 to .2 (power = .43) than for comparing .4 to .5 (power = .25).

Additional Issues in Sample Size Selection

- Adjustment for other factors/design issues in determining sample sizes—SD's after adjusting for confounders, general regression analyses, repeated measures analyses
- Multiple comparison issues
- Interim analyses

Planning the experiment

- Determine measurements of interest, hypotheses and methods for analysis
- Determine sample sizes needed so that power is as desired
  - Clinical vs statistical significance
- Different objectives/analyses require different power calculations
- Other approaches besides power calculations

Alternative approach to sample size selection

- Precision of estimators

  Want estimate of population mean, i.e. the sample mean, to be within

  d units of population mean with high probability, say .95

  equivalently

  Want 95% confidence interval to have width 2d.

  What sample size is needed?

More about Covariates

- Men and Women Combined? Ignore gender in design and analysis?
- Age?
- Initial Weight?
- Exercise level?

- What if everyone in new program exercised a lot and everyone in standard program exercised very little?
- Difference due to diet? Exercise?
- Great variability due to exercise?

- Important to consider potential confounding variables (such as exercise level)
  - At design stage
  - In analysis

Confounding Example:

- Average weight loss for men for each treatment: 10 lbs
- Average weight loss for women for each treatment: 5 lbs

- **No Difference** between treatments for each gender

- Group 1(Trt 1) has 15 women and 5 men.
- Group 2 (Trt 2) has 15 men and 5 women

- Anticipated observed difference between groups?

In general confounding can result in larger or smaller estimated treatment differences (than what you would see without confounding).

Note: Gender has an effect on weight loss and group is associated with gender

- Study from the United Kingdom recruiting subjects in 1972-1974 and obtaining smoking history (and other information) and following them up 20 years later.

- Look at data for 609 women aged 45-74 at start of study.

	Smoker	
	Yes	No
Dead	107	153
Alive	174	175
Total	281	328
%Dead	38.1%	46.6%

OR= $\frac{\left(\frac{107/281}{174/281}\right)}{\left(\frac{153/329}{175/329}\right)} = .703$ p-value=.033, Risk ratio=.82

	Age: 45-54		Age: 55-64		Age: 65-74	
	Smoker		Smoker		Smoker	
	Yes	No	Yes	No	Yes	No
Dead	27	12	51	40	29	101
Alive	103	66	64	81	7	28
%Dead	20.8%	15.4%	44.3%	33.1%	80.6%	78.3%

Confounding versus Interaction

Example of Interaction

- Mean reduction for standard program is 10 pounds for both genders
- Mean reduction for women on new diet = 15 lbs (n=10)
- Mean reduction for men on new diet =5 lbs (n=10)

  The effect of diet is different for men than for women
- Combining men and women (equal sample sizes) mean reduction for new program (ignoring gender) is 10 pounds
- Ignoring gender, would not expect to see a difference between diets.